TECHNICAL NOTE 31 - Practical Analysis of the Pierce Oscillator

To achieve optimum performance from a Pierce crystal oscillator, e.g. good frequency stability and low long-term aging, the crystal parameters, crystal drive current, and oscillator gain requirements must be carefully considered. Over many years of experience, it has been found that excessive crystal drive current is one of the main causes of oscillator malfunction. Overdriving the crystal causes frequency instability over time, and for tuning-fork crystals the excessive motional displacement can break the crystal tines.

This technical note describes a practical approach for measuring the key parameters of a Pierce Oscillator. This allows the designer to check the oscillator design against the actual oscillator performance and ensure that the oscillator design rules are met. (For analysis, see Statek Technical Note 30.) This note covers the following topics:

1. A practical procedure for determining the crystal drive current.
2. A practical procedure for measuring the amplifier's transconductance g_m and output resistance R_O.
3. A practical procedure for determining the total load capacitance C_L of the Pierce Oscillator circuit.

Basic Crystal Oscillator

The basic quartz crystal CMOS Pierce where oscillator circuit configuration is shown in Figure 1. The crystal oscillator circuit consists of an amplifying section and a feedback network. For oscillation to occur, the Barkhausen criteria must be met:

a) The loop gain must be greater than unity.
b) The phase shift around the loop must be an integer multiple of 2π.

The CMOS amplifier provides the amplification while the two capacitors C_D and C_G, the resistor R_A, and the crystal work as the feedback network. The resistor R_A stabilizes the output voltage of the amplifier and is used to reduce the crystal drive level.

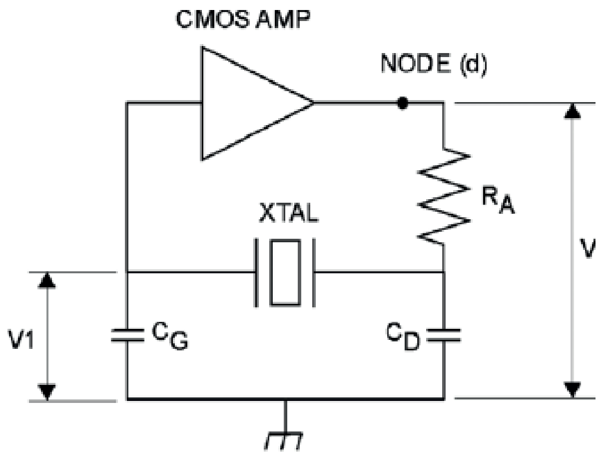

Figure 1 – Basic Pierce Oscillator Circuit

The crystal drive current is given by the equation

$$i_b = \left(\frac{\sqrt{\left(1+\frac{X_e}{X_0'}\right)^2 + \left(\frac{R_e}{X_0'}\right)^2}}{\sqrt{\left(R_e + R_A\left(1-\frac{X_e'}{X_0'}\right)\right)^2 + \left(X_e' + R_A\frac{R_e}{X_D}\right)^2}} \right) |V|,$$

where

$$X_0' = \frac{1}{\omega C_0'} = \frac{1}{\omega (C_0 + C_s)}.$$

The gain equation is

$$g_m \geq 4\pi^2 f^2 C_G \left[(C_D + C_d) R_e + \left(C_d + \frac{R_e}{R_O} C_d \right) R_A \right]$$
$$+ \frac{C_G}{C_D\left(1+\frac{R_A}{R_O}\right)+C_d} \left(4\pi^2 f^2 C_d C_D R_A + \frac{1}{R_O}\right)\left(1+\frac{R_A+R_e}{R_O} - 4\pi^2 f^2 C_d C_D R_A R_e\right)$$

The operating frequency is given by

$$f = f_s\left(1+\frac{C_1}{2(C_0+C_L)}\right),$$

where C_L is the load capacitance of oscillation. The equations for the oscillation frequency, gain and crystal drive are derived using a closed loop and phase diagram analysis of a CMOS quartz crystal oscillator. For more details see IQD Statek Technical Note 30.

Measuring the Crystal Drive Level

One of the most important parameters for a good oscillator design is the crystal drive current. With increasing demand for ultra-miniature quartz crystal resonators, the crystal parameters and the oscillator components must be carefully considered. The maximum recommended crystal drive level should not be exceeded.

The main factors affecting the drive level are the supply voltage (V_{DD}), R_1, R_A, C_D, C_G, and the stray capacitance, C_S.

Measurement Procedure

1. Measure C_G: Remove the crystal and neutralize node (d). With no power applied, measure C_G using a capacitance meter.
2. Measure C_D: Remove the crystal and disconnect R_A. With no power applied, measure C_D and C_d using a capacitance meter.
3. Calculate the impedances $X_G=1/(\omega C_G)$, $X_D=1/(\omega C_D)$, and $X_d = 1/(\omega C_d)$.
4. Mount the crystal, reconnect R_A and turnon the operating supply voltage.
5. Using a scope probe with no more than 2pF capacitance, measure the peak-to-peak voltages across C_G and C_D. Note: AC-couple the signal to the scope probe to avoid changing the DC bias voltage.
6. Calculate the peak-to-peak currents: $(I_G)_{p-p} = (V_G)_{p-p}/X_G$ and $(I_D)_{p-p} = (V_D)_{p-p}/X_D$.
7. Calculate the RMS currents:

$$(I_D)RMS = \frac{(I_D)_{P-P}}{2\sqrt{2}} \text{ and}$$

$$(I_G)RMS = \frac{(I_G)_{P-P}}{2\sqrt{2}}$$

8. The AC current through the crystal will be between $(I_G)_{RMS}$ and $(I_D)_{RMS}$, being closer to $(I_G)_{RMS}$.
9. The measured current should not exceed the maximum recommended value

$$I_{RMS} \le \sqrt{\frac{(\text{maximum allowed power})}{R_1}}$$

Measuring the Transconductance and Output Impedance of the Amplifier

For the oscillator to start up, the transconductance of the amplifier must be greater than the value given by the gain equation, and as a general rule it is best that it at least 2-3 times this minimum.

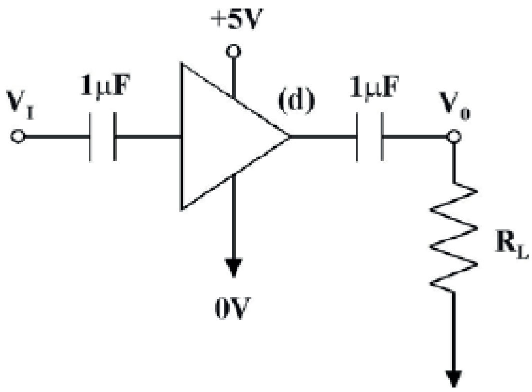

Figure 2 – Measuring the Transconductance and Output Impedance of the Amplifier

Measurement Procedure

1. Apply a sinusoidal voltage V_I to the input of the amplifier, coupled through a 1μF capacitor (with the voltage sufficiently low so that the output does not saturate). (See Figure 2.)

2. Measure the output voltage VO through a 1 μF capacitor.

3. Measure the output voltage with various load resistances to ground.

For example consider the sequence of measurements in Table 1.

Table 1: Measuring R_O

V_I [V]	R_L [kΩ]	V_O [V]
0.0168	OPEN	1.068
0.0168	30	0.636
0.0168	18	0.520
0.0168	15	0.464

The output resistance R_O of the amplifier is approximately equal to that load resistance R_L such that the output voltage V_O is one half of the output voltage when the load resistance is infinite (open). In the above example, at a load resistance R_L of 18 kΩ, the output voltage V_O is approximately equal to one-half of V_O when R_L is an open circuit. Therefore, R_O is equal to approximately 18kΩ.

The amplifier's transconductance g_m is the equal to the voltage gain divided by R_O.

$$g_m = (V_O / V_I) / R_O$$
$$= (1.068 / 0.0168) / (18\,\text{k}\Omega)$$
$$= 3.53\,\text{mS}.$$

The required minimum transconductance of the oscillator is calculated as follows:

1. Measure C_G and C_D, as described above.
2. Measure stray capacitance C_S across the crystal: Remove the crystal, neutralize the positive (V_{dd}) and ground, then measure the capacitance across the crystal termination.

Table 2: Oscillator Parameters

	Symbol	Value	Unit
Gate Capacitance	C_G	7.4	pF
Drain Capacitance	C_D	5.4	pF
Stray Capacitance	C_S	1.4	pF
Amp. Output capacitance	C_d	7.0	pF
Amp. Output resistance	R_O	70	kΩ
Limiting Resistance	R_A	30	kΩ

Table 3: Crystal Parameters

	Symbol	Value	Unit
Frequency	f_s	1.0	MHz
Motional resistance	R_1	3.0	KΩ
Shunt capacitance	C_O	1.2	pF

Using the gain equation, we find for the oscillator and crystal described in Tables 2 and 3

$$g_m \geq 0.119\ mS$$

The ratio of the amplifier's transconductance to the minimum transconductance required for oscillation is (3.53/0.119) = 29.7. Therefore, this circuit meets both the required minimum transconductance and the 2-3 times the minimum rule.

Measuring the Load Capacitance of the Oscillator Circuit

Properly specifying the load capacitance of the oscillator circuit allows the crystal manufacturer to tune the crystal frequency to the operating frequency of the oscillator. Given a crystal of known fs, C1, and C0, operating at a frequency f in a circuit, the load capacitance of the circuit is found from the frequency equation

$$C_L = \frac{C_1}{2}\left(\frac{f_s}{f - f_s}\right) - C_0$$

Measurement Procedure

1. Measure the crystal parameters C_1, C_0, and f_s with the use of a C_I meter or an impedance analyzer.
2. Install the measured crystal in the oscillator circuit and measure the oscillation frequency f.
3. Then calculate the load capacitance C_L.

For example with
f_s = 32.7644 kHz
C_1 = 2.3 fF
C_O = 1.5 pF
f = 32.768 kHz
we find C_L = 9.0 pF
The calculated load capacitance includes the stray capacitance across the crystal (C_s).

Glossary

L_1 Crystal Motional Inductance
C_1 Crystal Motional Capacitance
C_0 Crystal Shunt Capacitance
R_1 Crystal Motional Resistance
R_A Limiting Resistance
f_s Series Resonant Frequency of the Crystal
f Operating Frequency
V_I Input Voltage
V_O Output Voltage
C_L Total Load Capacitance of the Oscillator
C_s Total Stray Capacitance Across the Crystal
C_D Drain Capacitance
C_G Gate Capacitance
I_b Crystal Drive Current
g_m Transconductance
R_O Amplifier Output Resistance
C_d Amplifier Output Capacitance