

# TECHNICAL NOTE 31

## Practical Analysis of the Pierce Oscillator

### Introduction

To achieve optimum performance from a Pierce crystal oscillator, e.g. good frequency stability and low long term aging, the crystal parameters, crystal drive current, and oscillator gain requirements must be carefully considered. Over many years of experience, it has been found that excessive crystal drive current is one of the main causes of oscillator malfunction. Overdriving the crystal causes frequency instability over time, and for tuning-fork crystals the excessive motional displacement can break the crystal tines.

This technical note describes a practical approach to measuring the key parameters of a Pierce oscillator. This allows the designer to check the oscillator design against the actual oscillator performance and ensure that the oscillator design rules are met. (For analysis, see Statek Technical Note 30.) This note covers the following topics:

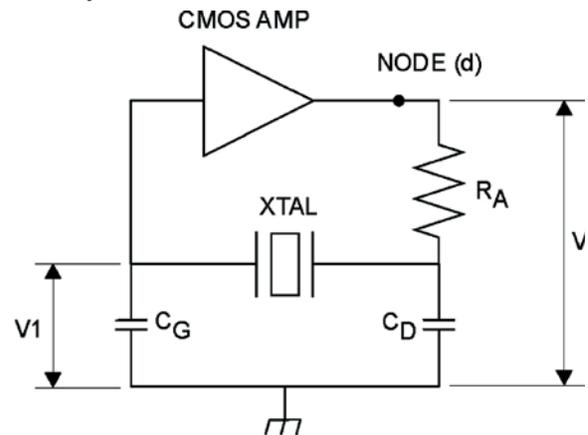
1. A practical procedure for determining the crystal drive current.
2. A practical procedure for measuring the amplifier's transconductance  $g_m$  and output resistance  $R_O$ .
3. A practical procedure for determining the total load capacitance  $C_L$  of the Pierce oscillator circuit.

### Basic Crystal Oscillator

The basic quartz crystal CMOS Pierce oscillator circuit configuration is shown in Figure 1. The crystal oscillator circuit consists of an amplifying section and a feedback network. For oscillation to occur, the Barkhausen criteria must be met:

- (a) The loop gain must be greater than unity.
- (b) The phase shift around the loop must be an integer multiple of  $2\pi$ .

The CMOS amplifier provides the amplification while the two capacitors  $C_D$  and  $C_G$ , the resistor  $R_A$ , and the crystal work as the feedback network. The resistor  $R_A$  stabilizes the output voltage of the amplifier and is used to reduce the crystal drive level.



**Figure 1:** Basic Pierce Oscillator Circuit

The crystal drive current is given by the equation

$$i_b = \left( \frac{\sqrt{\left(1 + \frac{X_e}{X'_0}\right)^2 + \left(\frac{R_e}{X'_0}\right)^2}}{\sqrt{\left(R_e + R_A \left(1 - \frac{X'_e}{X'_0}\right)\right)^2 + \left(X'_e + R_A \frac{R_e}{X_D}\right)^2}} \right) |V|,$$

where

$$X'_0 = \frac{1}{\omega C'_0} = \frac{1}{\omega(C_0 + C_s)}.$$

The gain equation is

$$g_m \geq 4\pi^2 f^2 C_G \left[ (C_D + C_d) R_e + \left( C_d + \frac{R_e}{R_O} C_d \right) R_A \right] + \frac{C_G}{C_D \left( \frac{R_A}{R_O} + C_d \right)} \left( 4\pi^2 f^2 C_d C_D R_A + \frac{1}{R_O} \right) \left( \frac{R_A + R_e}{R_O} - 4\pi^2 f^2 C_d C_D R_A R_e \right)$$

The operating frequency is given by

$$f = f_s \left( 1 + \frac{C_1}{2(C_0 + C_L)} \right),$$

where  $C_L$  is the load capacitance of oscillation. The equations for the oscillation frequency, gain and crystal drive are derived using a closed loop and phase diagram analysis of a CMOS quartz crystal oscillator. For more details see IQD Statek Technical Note 30.

### Measuring the Crystal Drive Level

One of the most important parameters for a good oscillator design is the crystal drive current. With increasing demand for ultra miniature quartz crystal resonators, the crystal parameters and the oscillator components must be carefully considered. The maximum recommended crystal drive level should not be exceeded.

The main factors affecting the drive level are the supply voltage ( $V_{DD}$ ),  $R_1$ ,  $R_A$ ,  $C_D$ ,  $C_G$ , and the stray capacitance,  $C_s$ .

#### Measurement Procedure

1. Measure  $C_G$ : Remove the crystal and neutralize node (d). With no power applied, measure  $C_G$  using a capacitance meter.
2. Measure  $C_D$ : Remove the crystal and disconnect  $R_A$ . With no power applied, measure  $C_D$  and  $C_d$  using a capacitance meter.
3. Calculate the impedances  $X_G = 1/(\omega C_G)$ ,  $X_D = 1/(\omega C_D)$ , and  $X_d = 1/(\omega C_d)$ .
4. Mount the crystal, reconnect  $R_A$  and turn on the operating supply voltage.
5. Using a scope probe with no more than 2 pF capacitance, measure the peak-to-peak voltages across  $C_G$  and  $C_D$ . Note: AC-couple the signal to the scope probe to avoid changing the DC bias voltage.

6. Calculate the peak-to-peak currents:  $(I_G)_{p-p} = (V_G)_{p-p}/X_G$  and  $(I_D)_{p-p} = (V_D)_{p-p}/X_D$ .

7. Calculate the RMS currents:

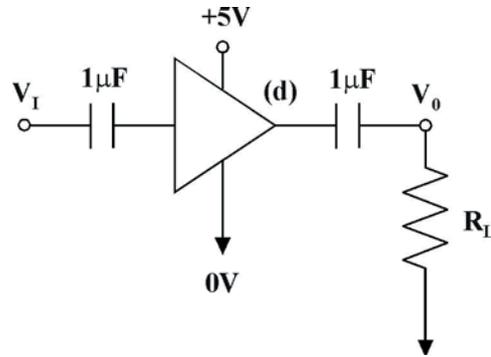
$$(I_D)_{RMS} = (I_D)_{p-p}/(2\sqrt{2}) \text{ and } (I_G)_{RMS} = (I_G)_{p-p}/(2\sqrt{2})$$

8. The AC current through the crystal will be between  $(I_G)_{RMS}$  and  $(I_D)_{RMS}$ , being closer to  $(I_G)_{RMS}$ .
9. The measured current should not exceed the maximum recommended value

$$I_{RMS} \leq \sqrt{\frac{\text{(maximum allowed power)}}{R_1}}$$

### Measuring the Transconductance and Output Impedance of the Amplifier

For the oscillator to start up, the transconductance of the amplifier must be greater than the value given by the gain equation, and as a general rule it is best that it at least 2-3 times this minimum.



**Figure 2:** Measuring the Transconductance and Output Impedance of the Amplifier

#### Measurement Procedure

1. Apply a sinusoidal voltage  $V_I$  to the input of the amplifier, coupled through a 1  $\mu$ F capacitor (with the voltage sufficiently low so that the output does not saturate). (See Figure 2.)
2. Measure the output voltage  $V_O$  through a 1  $\mu$ F capacitor.

3. Measure the output voltage with various load resistances to ground.

For example consider the sequence of measurements in Table 1.

**Table 1:** Measuring  $R_o$

$V_i$ [V]	$R_L$ [k $\Omega$ ]	$V_o$ [V]
0.0168	OPEN	1.068
0.0168	30	0.636
0.0168	18	0.520
0.0168	15	0.464

The output resistance  $R_o$  of the amplifier is approximately equal to that load resistance  $R_L$  such that the output voltage  $V_o$  is one half of the output voltage when the load resistance is infinite (open). In the above example, at a load resistance  $R_L$  of 18 k $\Omega$ , the output voltage  $V_o$  is approximately equal to one-half of  $V_o$  when  $R_L$  is an open circuit. Therefore,  $R_o$  is equal to approximately 18 k $\Omega$ .

The amplifier's transconductance  $g_m$  is the equal to the voltage gain divided by  $R_o$ .

$$\begin{aligned} g_m &= (V_o / V_i) / R_o \\ &= (1.068 / 0.0168) / (18 \text{ k}\Omega) \\ &= 3.53 \text{ mS}. \end{aligned}$$

The required minimum transconductance of the oscillator is calculated as follows:

1. Measure  $C_G$  and  $C_D$ , as described above.
2. Measure stray capacitance  $C_s$  across the crystal: Remove the crystal, neutralize the positive ( $V_{dd}$ ) and ground, then measure the capacitance across the crystal termination.

**Table 2:** Oscillator Parameters

	Symbol	Value	Unit
Gate capacitance	$C_G$	7.4	pF
Drain capacitance	$C_D$	5.4	pF
Stray capacitance	$C_s$	1.4	pF
Amp. Output capacitance	$C_d$	7.0	pF
Amp. Output resistance	$R_o$	70	k $\Omega$
Limiting resistance	$R_A$	30	k $\Omega$

**Table 3:** Crystal Parameters

	Symbol	Value	Unit
Frequency	$f_s$	1.0	MHz
Motional resistance	$R_1$	3.0	K $\Omega$
Shunt capacitance	$C_0$	1.2	pF

Using the gain equation, we find for the oscillator and crystal described in Tables 2 and 3

$$g_m \geq 0.119 \text{ mS}$$

The ratio of the amplifier's transconductance to the minimum transconductance required for oscillation is  $(3.53/0.119) = 29.7$ . Therefore, this circuit meets both the required minimum transconductance and the 2-3 times the minimum rule.

### Measuring the Load Capacitance of the Oscillator Circuit

Properly specifying the load capacitance of the oscillator circuit allows the crystal manufacturer to tune the crystal frequency to the operating frequency of the oscillator. Given a crystal of known  $f_s$ ,  $C_1$ , and  $C_0$ , operating at a frequency  $f$  in a circuit, the load capacitance of the circuit is found from the frequency equation

$$C_L = \frac{C_1}{2} \left( \frac{f_s}{f - f_s} \right)^2 - C_0$$

### Measurement Procedure

1. Measure the crystal parameters  $C_1$ ,  $C_0$ , and  $f_s$  with the use of a CI meter or an impedance analyzer.
2. Install the measured crystal in the oscillator circuit and measure the oscillation frequency  $f$ .
3. Then calculate the load capacitance  $C_L$ .

For example with

$$\begin{aligned} f_s &= 32.7644 \text{ kHz} \\ C_1 &= 2.3 \text{ fF} \\ C_0 &= 1.5 \text{ pF} \\ f &= 32.768 \text{ kHz} \end{aligned}$$

we find

$$C_L = 9.0 \text{ pF}.$$

The calculated load capacitance includes the stray capacitance across the crystal ( $C_s$ ).

## Glossary

**Table 4: Glossary of Symbols**

$L_1$	Crystal Motional Inductance
$C_1$	Crystal Motional Capacitance
$C_0$	Crystal Shunt Capacitance
$R_1$	Crystal Motional Resistance
$R_A$	Limiting Resistance
$f_s$	Series Resonant Frequency of the Crystal
$f$	Operating Frequency
$V_I$	Input Voltage
$V_O$	Output Voltage
$C_L$	Total Load Capacitance of the Oscillator
$C_s$	Total Stray Capacitance Across the Crystal
$C_D$	Drain Capacitance
$C_G$	Gate Capacitance
$i_b$	Crystal Drive Current
$g_m$	Transconductance
$R_O$	Amplifier Output Resistance
$C_d$	Amplifier Output Capacitance